LEXINGTON CHRISTIAN

LEAGUES

MAML

STATE INVITATIONAL MATH LEAGUE COMPETITION

March 31, 2017

Shrewsbury High School

MASSACHUSETTS ASSOCIATION OF MATHEMATICS LEAGUES

STATE PLAYOFFS – 2017

Round 1 **Arithmetic and Number Theory**

1. ____________________

2. ____________________

3. ____________________

1. Three-fourths of a cup of PowerMax Cereal gives Tom his daily value of potassium. Yesterday, Tom had $\frac{1}{3}$ cup of PowerMax in the morning and $\frac{1}{5}$ cup at lunch. What fraction of Tom's daily value of potassium did he get yesterday from PowerMax?

2. Compute the number of ordered triples (a,b,c), where a, b, and c are positive integers such that $a^3+b^3+c^3=153$.

3. A combination lock for a bicycle consists of four cylinders, each of which has all the whole numbers from 1 to 6. Alvin forgot his combination so he tried each number in order, starting with 1111, then 1112, followed by 1113, and so on until the lock finally opened at 2563. Compute the number of unsuccessful lock combinations that he tried.

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Round 2 **Algebra 1**

1. ____________________

2. ____________________

3. ____________________

1. Compute the largest possible product xy of the ordered pair solutions (x, y) to the system:

$$x^2 + y = 8 \text{ and } y - x = -4.$$

2. If $(a+b):(b+c):(a+c) = 1:2:5$ and $a+b+c = 40$, compute the value of a.

3. Compute all real solutions to $\sqrt{|x-1|-1} = \sqrt{|x+1|+1}$.

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Round 3 – Geometry

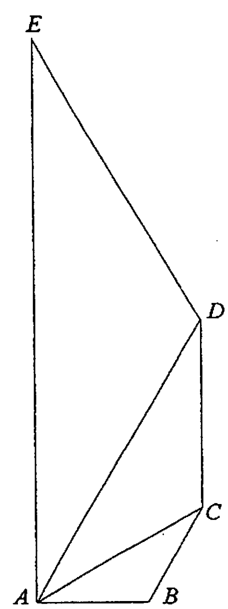
1. ____________________

2. ____________________

3. ____________________

1. Circles P and Q are externally tangent. Circle P has a radius of 4 units and circle Q has a radius of 8 units. Radius $\overline{MP}$ and radius $\overline{RQ}$ are perpendicular to $\overline{PQ}$. Compute the number of square units in the area of ΔMPR.

2. Shown are three similar triangles with angle 30° and 120°. Find the ratio of the area of ΔABC to the area of $ABCDE$.



3. The lengths of three segments are $x^2 + 3x$, $x^2 + x$, and 16. Find all values of x such that when the three segments are joined at their endpoints, a triangle is formed.

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Round 4 – Algebra 2

1. ______________________

2. ______________________

3. ______________________

1. If $3x^2 + bx + 20$ is factorable over the integers, how many possible values of b are there?

2. A famous English palindromic sentence is: A man, a plan, a canal, Panama! Compute the number of distinct 21-letter palindromic 'words' that can be formed from the letters of that sentence, namely from **amanaplanacanalpanama**.

3. Given the system $5x+3y+4z=31, 3x+y+2z=15$ and $x+y+z=8$, compute all possible ordered triples (x,y,z) over the set of positive integers. (Proper ordered triple notation must be used in your answer.)

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Round 5 – Analytic Geometry

1. ______________________

2. ______________________

3. (__________ , __________)

1. The graphs of $9x^2 + 4y^2 = 36$ and $y = \pm|2x|$ intersect in 4 points. Find the number of square units in the area of the quadrilateral whose vertices are those 4 points.

2. A line with negative slope passes through (4, 5). The x-intercept is four times the y-intercept. Compute the value of x, if the point (x, 18) lies on the line.

3. A rectangle is inscribed in a parabola whose equation is $y = b - ax^2$. Two vertices of the rectangle lie on the x-axis. The dimensions of the rectangle with the largest perimeter are 10 by 23, where the side of length 10 lies on the x-axis. Compute the ordered pair (a, b).

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Round 6 – Trig and Complex Numbers

1. ____________________

2. ____________________

3. ____________________

1. Determine $\cos\left(\mathrm{Sin}^{-1}\left(-\frac{1}{2}\right)+\mathrm{Tan}^{-1}(1)\right)$

2. In ΔABC, $A=(0,0)$, $B=(\sin a,\cos a)$, and $C=(\cos b,\sin b)$, where a and b are in radians with $0<a<b<\frac{\pi}{4}$. If the area of ΔABC is 1/10, compute the tangent of $\angle BAC$.

3. The function $r=\dfrac{4}{2\cos\theta+\sin\theta}$ is in polar coordinates. Compute the least possible positive value of r that the function takes on.

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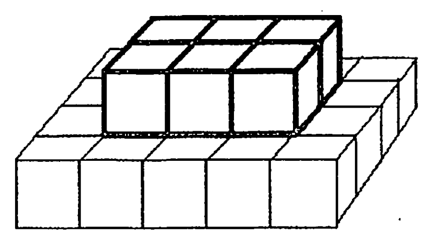
Team Round

Place answers on Team Round Answer Sheet

1. Circles P and Q are externally tangent at T. At time $t = 0$, points A and B are both at T. Then, at the same time, A and B start moving, A traveling counterclockwise on P and B traveling clockwise on Q. If A makes one revolution every 10 seconds and B makes one revolution every 16 seconds, compute the least possible positive value of t (in seconds) at which points P, Q, A, and B are collinear.

2. The lengths of the sides of a triangle are $2a$, $4b$, and $7a$ with $2a < 4b < 7a$. Determine the least integer value of a such that there are 50 integer values of b that yield a non-degenerate triangle.

3. Using sticky toothpicks, a student builds a structure consisting of a base of 20 cubes in a 5 by 4 arrangement and a second layer of 6 cubes in a 2 by 3 arrangement. The top layer has edges in common with the bottom as indicated in the diagram. Whenever edges are in common, only 1 toothpick was used. Compute the number of toothpicks that were used. Note: The diagram doesn't show all the toothpicks.



4. Let h be greater than 0 and not equal to 2. The graphs of $y = x^2$ and $y = (x-2)^2$ intersect at point A, the graphs of $y = (x-2)^2$ and $y = (x-h)^2$ intersect at B, and the graphs of $y = x^2$ and $y = (x-h)^2$ intersect at C. If $CA = CB$, compute the sum of all possible values of h.

5. Let $\lfloor x \rfloor$ = the greatest integer less than or equal to x. For $2 \le a \le 4$, compute the least possible value of $\lfloor \log_a(8a) + \log_{8a}(64a) \rfloor$

6. Let S be the set of positive 4-digit numbers none of whose digits are 0. A number is chosen at random from S. Compute the probability that at least two adjacent digits are the same.

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STATE PLAYOFFS – 2017 – ANSWER SHEET

Round 1

1. $\frac{32}{45}$
2. 6
3. 392

Round 2

1. 32
2. 20
3. $x \le -1$

Round 3

1. 24
2. $\frac{1}{13}$
3. $-8 < x < -4$ or $2 < x < 8$ or $(-8,-4) \cup (2,8)$

Round 4

1. 12
2. 15120
3. $(1,2,5)$, $(2,3,3)$, and $(3,4,1)$

Round 5

1. $\frac{288}{25}$
2. -48
3. $\left(\frac{1}{5}, 28\right)$

Round 6

1. $\frac{\sqrt{6}+\sqrt{2}}{4}$ or $\frac{1}{2}\sqrt{2+\sqrt{3}}$
2. $\frac{\sqrt{6}}{12}$
3. $\frac{4\sqrt{5}}{5}$

Team

1. 40
2. 101
3. 157
4. $2 + 2\sqrt{2}$
5. 4
6. $\frac{217}{729}$

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Solutions: State Meet 2017

Round 1 Arithmetic and Number Theory:

1. $\frac{3}{4}:1=\left(\frac{1}{3}+\frac{1}{5}\right):x \rightarrow \frac{3}{4}x=\frac{8}{15} \rightarrow$ $x=$ $\underline{\mathbf{\frac{32}{45}}}$

2. Note that the numbers must be less than 6.

   Suppose the largest of the three numbers is 5.

   If $a=5$, then $b^3+c^3=28$ and only 1 and 3 satisfy that equation.

   All six permutations of (5, 1, 3) satisfy the equation.

   Suppose the largest number is 4.

   If $a=4$, then $b^3+c^3=89$ and, for b = 1, 2, and 3, c is not an integer.

   If all the numbers were 3, the largest possible value of $a^3+b^3+c^3$ would be 81and this case fails.

   Thus, the total number of ordered triples is **6**.

3. In base 6,
   1111 to 2110 is $2110-1111+1=1000_6$ combinations.
   2111 to 2510 is $2510-2111+1=400_6$ combinations.
   2511 to 2562 is $2562-2511+1=52_6$ combinations.
   Converting to base 10, 216 + 144 + 32 = **392**.

Round 2 Algebra I:

1. Rewrite the second equation as $x-y=4$ and add to the first equation, obtaining $x^2+x-12=0$.
   From $(x+4)(x-3)=0$, we obtain $x=-4$ or 3, giving $(x,y)=(-4,-8),(3,-1)$.
   The largest possible product is **32**.

2. From $\frac{a+b}{b+c}=\frac{1}{2}$, we obtain $2a+2b=b+c$. Adding a to both sides, $3a+2b=a+b+c=40$.

   From $\frac{a+b}{a+c}=\frac{1}{5}$, we obtain $5a+5b=a+c$. Adding b to both sides, $5a+6b=a+b+c=40$.

   Thus, we have the system $3a+2b=40$ and $5a+6b=40$.

   Multiplying the first equation by 3 and subtracting, gives us $4a=80$, so a = **20**.

3. First check the domain of the equation.
The right side is defined for all x.
However, on the left side we have $|x-1| \geq 1 \rightarrow (x-1 \leq -1 \text{ or } x-1 \geq 1) \leftrightarrow \boxed{x \leq 0 \text{ or } x \geq 2}$.
Squaring both sides, $|x-1|-1 = |x+1|+1$.
The critical values are ± 1.
Replace the absolute value expressions according to these three cases:

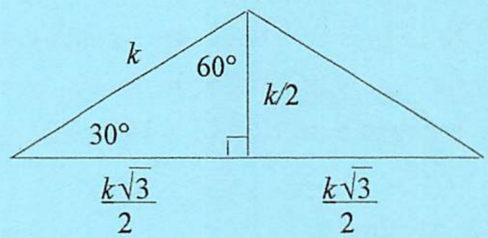
i) $x \geq 1 \rightarrow x-1-1 = x+1+1 \rightarrow -2 = 2$. There are no solutions in this interval.
ii) $-1 < x < 1 \rightarrow 1-x-1 = x+1+1 \rightarrow -2 = 2x \rightarrow x = -1$. No solutions in this interval.
iii) $x \leq -1 \rightarrow 1-x-1 = -x-1+1 \rightarrow 0 = 0$.
This is true for all x in this interval, giving $x \leq -1$ for the solution set.

Since all the solutions in (iii) lie in the domain, the answer is $\underline{\boldsymbol{x \leq -1}}$.

Round 3 Geometry:

1. Area ΔMPR = Area trapezoid $MPQR$ – Area PQR. Area trapezoid $MPQR = \frac{1}{2} \cdot 12 \cdot (8+4) = 72$

Area $PQR = \frac{1}{2} \cdot 12 \cdot 8 = 48$. $\therefore$ Area $\Delta MPR = \underline{\mathbf{24}}$



2. As the diagram shows, in a 30-120-30 triangle the ratio of the long side to the short side is $\sqrt{3}:1$.
Thus, the ratio of the area of ACD to the area of ABC is 3:1. Likewise, the ratio of the area of ADE to ACD is 3:1, so the ratio of ADE to ABC is 9 : 1. Finally,

$$\frac{a(\Delta ABC)}{a(ABCDE)} = \frac{a(\Delta ABC)}{a(\Delta ABC)+a(\Delta ACD)+(\Delta ADE)} = \frac{a(\Delta ABC)}{a(\Delta ABC)+3a(\Delta ABC)+9a(\Delta ABC)} = \underline{\mathbf{\frac{1}{13}}}.$$

3. Since the side lengths must be positive, we have $x^2+3x > 0 \rightarrow x < -3$ or $x > 0$.
Similarly, $x^2+x > 0 \rightarrow x < -1$ or $x > 0$. From the triangle inequality we obtain

(1) $(x^2+3x)+(x^2+x) > 16 \rightarrow x^2+2x-8 > 0 \rightarrow x < -4$ or $x > 2$.

(2) $\left(x^2 + x\right) + 16 > x^2 + 3x \rightarrow x < 8$.

(3) $\left(x^2 + 3x\right) + 16 > x^2 + x \rightarrow x > -8$.

The intersection of the four solution sets is $\underline{-8 < x < -4 \text{ or } 2 < x < 8}$.

This could also be written as $\underline{(-8,-4) \cup (2,8)}$.

Round 4 Algebra II

1. If the factored form is considered to be $(3x + m)(x + n))$, then $mn = 20$. There are three pairs of integers whose product is twenty: 1, 20; 2, 10; and 4, 5. The integers in each pair could be used for either *m* or *n* and both could be negative. This leads to the possible values of *b* to be 16, 17, 19, 23, 32, or 61 and their opposites. There are **12** possibilities.

2. Here *c* goes in the middle, leaving the letters *aaaaamnnpl* on either side, giving $\frac{10!}{5! \cdot 2!} = \mathbf{15120}$ different 'words'.

3. Subtracting the third equation from the second gives $2x + z = 7 \rightarrow z = 7 - 2x$. Let $x = t$ and $z = 7 - 2t$. Substituting into either of the original equations gives $y = t + 1$. From $x = t$, we obtain $t > 0$ and, from $z = 7 - 2t$, we obtain $7 - 2t > 0 \rightarrow t < 3.5$. Thus, *t* can take on the values of 1, 2, and 3, giving 3 ordered triples of positive integers.
   Those triples are $\mathbf{(1,2,5),(2,3,3)}$, and $\mathbf{(3,4,1)}$.
   If the second equation is subtracted from the first and the result divided by 2, the third equation is the result. Then follow above. The system of equations does not produce a unique solution because the equations are not independent.

Round 5 Analytic Geometry:

1. Substituting $y = \pm|2x|$ into the equation of the ellipse gives $9x^2 + 16x^2 = 36 \rightarrow x = \pm\frac{6}{5}$.
   Then $y = \pm\frac{12}{5}$. The area is $4 \cdot \frac{6}{5} \cdot \frac{12}{5} = \mathbf{\frac{288}{25}}$.

2. Let the y-intercept equal b. Using the intercept form for the equation of a line, we have $\frac{x}{4b}+\frac{y}{b}=1$. If (4, 5) lies on the line, we have $\frac{4}{4b}+\frac{5}{b}=1 \to \frac{1}{b}+\frac{5}{b}=1$, so $b=6$.

The equation of the line is $\frac{x}{24}+\frac{y}{6}=1$ and , when $y=18$, $\frac{x}{24}=-2$, so $x=\underline{\mathbf{-48}}$.

3. The perimeter P equals $4x+2\left(b-ax^2\right)=-2ax^2+4x+2b = -2a\left(x^2-\frac{2x}{a}\right)+2b =$

$-2a\left(x^2-\frac{2x}{a}+\frac{1}{a^2}\right)+2b+\frac{2}{a} = -2a\left(x-\frac{1}{a}\right)^2+\frac{2ab+2}{a}$. The maximum will occur at $x=\frac{1}{a}$, and since the width is $2x=\frac{2}{a}$, we have $\frac{2}{a}=10$, so $a=\frac{1}{5}$. The height of the rectangle is the y-value of the parabola at $x=\frac{1}{a}$ so $y=b-a\cdot\frac{1}{a^2}=b-\frac{1}{a}$. Thus, $23=b-5$. Answer: $\underline{\left(\mathbf{\frac{1}{5}, 28}\right)}$.

Round 6 Trigonometry and Complex Numbers:

1. $\text{Sin}^{-1}\left(-\frac{1}{2}\right)=-\frac{\pi}{6}$; $\text{Tan}^{-1}1=\frac{\pi}{4}$; $\cos\left(-\frac{\pi}{6}+\frac{\pi}{4}\right)=\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}-\left(-\frac{1}{2}\right)\cdot\frac{\sqrt{2}}{2}=\underline{\mathbf{\frac{\sqrt{6}+\sqrt{2}}{4}}}$.

2. The area of ΔABC is $\frac{1}{2}\cdot AB\cdot AC\cdot\sin\angle BAC$. $AB=\sqrt{\sin^2 a+\cos^2 a}=1$. Likewise, $AC=1$.

Thus, $\frac{1}{10}=\frac{1}{2}\cdot 1\cdot 1\cdot\sin\angle BAC \to \sin\angle BAC=\frac{1}{5}$. Then $\cos\angle BAC=\frac{\sqrt{24}}{5}=\frac{2\sqrt{6}}{5}$, making

$\tan\angle BAC=\frac{1}{2\sqrt{6}} = \underline{\mathbf{\frac{\sqrt{6}}{12}}}$.

3. Convert $r=\frac{4}{2\cos\theta+\sin\theta}$ to rectangular form: $r=\frac{4}{\frac{2x}{r}+\frac{y}{r}} \to r=\frac{4r}{2x+y} \to$

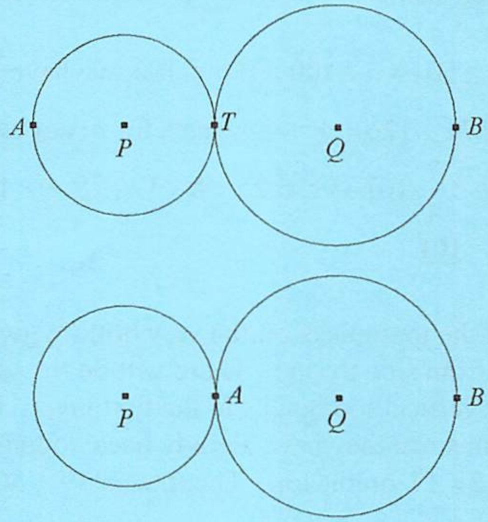
$1=\frac{4}{2x+y} \to 2x+y=4$. Using the distance formula from (0, 0) to the line,

$d=\frac{|2\cdot 0+1\cdot 0-4|}{\sqrt{2^2+1^2}} = \frac{4}{\sqrt{5}} = \underline{\mathbf{\frac{4\sqrt{5}}{5}}}$.

MASSACHUSETTS ASSOCIATION OF MATHEMATICS LEAGUES

Solutions: State Meet 2017

Team Round:



1. A makes 1/10 revolution per second and B makes 1/16 revolution per second. When points P, Q, A, and B are collinear, there are 3 possibilities:
   i) A and B are both back at T,
   ii) A and B are outside P and Q, or
   iii) one of A and B is at T and the other is outside. Cases (ii) and (iii) are shown in the diagram.

In (i), A and B have both made an integer number of revolutions; the difference in the number of revolutions is also an integer. In (ii) each has made half a revolution more than an integral number but the difference in the number of revolutions is still an integer. In (iii) one has made an integral number while the other has made an integer and a half revolution more so the difference in the number of revolutions involves half a revolution. Since we must allow for any case, we have $\frac{1}{10}t - \frac{1}{16}t = \frac{n}{2}$.

This simplifies to $n = \frac{3t}{40}$, so $t = \underline{\mathbf{40}}$ is the least value of t at which the points are collinear.

At 40 seconds, A has made 4 complete revolutions and B has made 2.5.

2. There are 3 inequalities that need to be satisfied. First, $4b > 2a \to b > \frac{a}{2}$. Second, $4b < 7a \to b < \frac{7}{4}a$. Third, by the triangle inequality, $2a + 4b > 7a \to 4b > 5a \to b > \frac{5}{4}a$. We choose the most restrictive inequalities, namely $\frac{5}{4}a < b < \frac{7}{4}a$. For there to be 50 integer values of b, we need $\frac{7}{4}a - \frac{5}{4}a \geq 50 \to a \geq 100$. If $a = 100$, we have $125 < b < 175$, giving $126 \leq b \leq 174$, yielding $174 - 126 + 1 = 49$ values for b, not enough. If $a = 101$, then $\frac{5}{4} \cdot 101 < b < \frac{7}{4} \cdot 101 \to 126.25 < b < 176.75 \to 127 \leq b \leq 176$, giving $176 - 127 + 1 = 50$ values for b. Thus, $a =$ **<u>101</u>**.

3. Counting the toothpicks on the very bottom gives $5 \cdot 5 + 6 \cdot 4 = 49$, as we count first from front to back and then from side to side. There will be the same number on the top of the first layer. There will be $6 \cdot 5 = 30$ vertical toothpicks in the first layer. For the second layer, we don't count the toothpicks on the bottom since they have already been counted. We have $3 \cdot 4 = 12$ vertical toothpicks and $3 \cdot 3 + 4 \cdot 2 = 17$ on the top. The total is 49 + 30 + 49 + 12 + 17 = **<u>157</u>**.

4. There are several cases to consider:

   1) For A, $x^2 = x^2 - 4x + 4 \to x = 1$, giving $A = (1,1)$.

   2) For B, $(x-2)^2 = (x-h)^2 \to x - 2 = h - x \to x = \frac{h+2}{2} = \frac{h}{2} + 1$. Then $B = \left(\frac{h}{2} + 1, \left(\frac{2-h}{2}\right)^2\right)$.

   3) For C, $x^2 = (x-h)^2 \to 2xh = h^2 \to x = \frac{h}{2}$. Then $C = \left(\frac{h}{2}, \frac{h^2}{4}\right)$.

   $AC = CA \to \sqrt{\left(\frac{h}{2} - 1\right)^2 + \left(\frac{h^2}{4} - 1\right)^2} = \sqrt{\left(\frac{h}{2} + 1 - \frac{h}{2}\right)^2 + \left(\frac{h^2}{4} - \left(\frac{4 - 4h + h^2}{4}\right)\right)^2}$. Squaring and simplifying gives $\frac{h^2}{4} - h + 1 + \frac{h^4}{16} - \frac{h^2}{2} + 1 = 1 + \left(1 - 2h + h^2\right) \to h^4 - 20h^2 + 16h = 0$.

   Since $h > 0$, we consider the cubic $h^3 - 20h + 16 = 0$. The Rational Root Theorem suggests that 4 could be a solution and we find that it is, giving $(h-4)\left(h^2 + 4h - 4\right) = 0$. The quadratic's solutions are $-2 \pm 2\sqrt{2}$. The positive values for h are 4 and $-2 + 2\sqrt{2}$. Their sum is **<u>$2 + 2\sqrt{2}$</u>**.

5. If $a=2$, we have $\log_2 16+\log_{16}128=5.75$. If $a=4$, we have $\log_4 32+\log_{32}256=\frac{41}{10}$, so 4 seems to be the answer. Can we obtain a result less than 4? Using the AM-GM, we have

$$\frac{\frac{\ln(8a)}{\ln a}+\frac{\ln(64a)}{\ln(8a)}}{2}\geq\sqrt{\frac{\ln(8a)}{\ln a}\cdot\frac{\ln(64a)}{\ln(8a)}}=\sqrt{\frac{\ln 64+\ln a}{\ln a}}=\sqrt{\frac{\ln 64}{\ln a}+1}.$$

Clearly, $\sqrt{\frac{\ln 64}{\ln a}+1}$ is least for the largest value of a. Letting $a=4$ gives $\log_a(8a)+\log_{8a}(64a)\geq 2\sqrt{\frac{\ln 4^3}{\ln 4}+1}$. Since $2\sqrt{\frac{\ln 4^3}{\ln 4}+1}=2\sqrt{3+1}=4$, $\log_a(8a)+\log_{8a}(64a)$ can't drop below 4, so the least possible value of $\lfloor\log_a(8a)+\log_{8a}(64a)\rfloor$ is **4**.

6. Solution #1: The Indirect Approach

Let the number be *ABCD*. Let's count the number of cases where no two adjacent digits are equal. Since none of digits can be 0, there are 9 choices for *A*. *B* must be different from *A* so there are 8 choices for *B*. *C* must be different from *B*, but it could equal *A*, so there are 8 choices for *C*. *D* must be different from *C*, but it could equal either *A* or *B*, so there are 8 choices for *D*. Thus, there are $9\cdot 8\cdot 8\cdot 8$ cases, where no two adjacent numbers are the same. There are 9^4 four-digit numbers, if 0 is not used, so the probability that no two adjacent numbers are equal is $\frac{9\cdot 8^3}{9^4}=\frac{8^3}{9^3}$. Thus, the probability that at least two adjacent numbers are the same is $1-\frac{8^3}{9^3}=$ $\mathbf{\frac{217}{729}}$.

Solution #2: The Direct Approach (Brute Force)

We'll look at all the possible cases, where at least 2 adjacent numbers are equal. There are 9^4 possible numbers. These are the possible cases:

1) *AABC*, *BAAC*, or *BCAA*. There are $3\cdot 9\cdot 8\cdot 7$ ways to do that.

2) *AABB* or *BBAA*. There are $2\cdot 9\cdot 1\cdot 8\cdot 1$ such numbers.

3) *AAAB* or *BAAA*. There are $2\cdot 9\cdot 1\cdot 1\cdot 8$ ways to do that.

4) *AAAA*. There are 9 such numbers.

5) *ABAA* or *AABA*. There are $2\cdot 9\cdot 1\cdot 1\cdot 8$ ways to do that.

The probability is $\frac{3\cdot 9\cdot 8\cdot 7+2\cdot 9\cdot 8+2\cdot 9\cdot 8+9+2\cdot 9\cdot 8}{9^4}=\frac{3\cdot 8\cdot 7+4\cdot 8+1+2\cdot 8}{9^3}=$ $\mathbf{\frac{217}{729}}$.