WORCESTER ACADEMY

LEAGUES

MAML

STATE INVITATIONAL MATH LEAGUE COMPETITION

April 4, 2014

Shrewsbury High School

MASSACHUSETTS ASSOCIATION OF MATHEMATICS LEAGUES

STATE PLAYOFFS – 2014

Round 1 **Arithmetic and Number Theory**

1. ____________________

2. ____________________

3. ____________________

1. Compute the <u>number</u> of ordered triples (a,b,c) where a, b, and c are positive integers such that $a^3+b^3+c^3=153$.

2. Let the factors, $a_1, a_2, \ldots a_9$ of a_9 be written in a square 3 by 3 array as indicated. Let $a_i < a_{i+1}$ for $i = 1, 2, \ldots, 8$. If $a_1 = 1$ and $a_6 = 117$, compute the value of a_8 .

$$\begin{matrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{matrix}$$

3. Let x equal the positive base 10 four-digit number $ABCD$, with $A, B \neq 0$, and let y equal $BCDA$. If $x < y$ and $x + y = 6897$, determine the <u>number</u> of ordered pairs (x, y).

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Round 2 **Algebra 1**

1. ______________________

2. ______________________

3. ______________________

1. If $a+b=3$ and $a^3+b^3=126$, compute a^2+b^2.

2. If $(a+b):(b+c):(a+c)=1:2:5$ and $a+b+c=40$, compute the value of a.

3. Compute all real solutions to $\sqrt{|x-1|-1}=\sqrt{|x+1|+1}$.

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Round 2 Algebra I

1. ______________________

2. ______________________

3. ______________________

1. If $a+b=3$ and $a^3+b^3=126$, compute a^2+b^2.

2. If $(a+b):(b+c):(a+c)=1:2:5$ and $a+b+c=40$, compute the value of a.

3. Compute all real solutions to $\sqrt{|x-1|-1}=\sqrt{|x+1|+1}$.

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Round 3 – Geometry

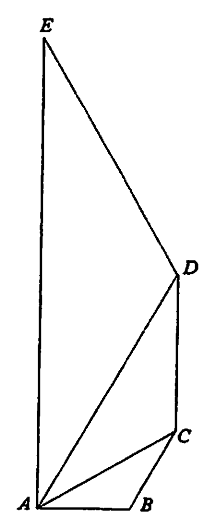
1. ______________________

2. ________:____________

3. ________:____________

1. ABC is an isosceles triangle with $AB = AC$ and $m\angle B = 72°$. $\overline{BA}$ is extended to E making line segment $\overline{BAE}$. If $\angle EAC$ is trisected and the trisector meets $\overleftrightarrow{BC}$ at D, making line segment $\overline{BCD}$, compute the degree measure of $\angle CDA$.

2. Shown are three similar triangles with angles of 30° and 120°. Find the ratio of the area of ΔABC to the area of $ABCDE$.



3. A small square lies wholly inside a large square without touching the sides. If the ratio of the perimeter of the small square to the perimeter of the large square is equal to the ratio of the area of the small square to the area of the region inside the large square and outside the small square, compute the ratio of the length of a side of the large square to the length of a side of the small square.

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Round 4 – Algebra 2

1. ______________________

2. ______________________

3. ______________________

1. Set S consists of five positive integers that form an increasing arithmetic sequence with a common difference of 20. Compute the least possible value of the median of S so that any three of the numbers can form the lengths of a triangle.

2. Find the sum of all complex values of x (complex includes real) that satisfy the following system:

$$x + xyz = 7$$
$$y + xyz = 8$$
$$z + xyz = 9$$

3. Let a, b, and c be real numbers with $a \le b \le c$. Compute all ordered triples (a,b,c) such that the product of any two of the numbers is 3 more than the sum of that pair.

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Round 5 – Analytic Geometry

1. (,)

2. ______________

3. ______________

1. The vertex of a parabola is at the center of the graph of $\frac{(x-3)^2+(y+2)^2}{25}=1$. The y-intercepts of the graph are also on the parabola. Find the coordinates of the focus of the parabola.

2. Compute the area bounded by the x-axis and the vertical and slant asymptotes of $y=\frac{x^2+2x-6}{x-2}$. .

3. Let $f(x)=\begin{cases} 3x+12 & -4\le x\le -2 \\ -3x & -2<x<0 \\ -2x & 0\le x\le 1 \\ -2 & 1<x<3 \\ 2x-8 & 3\le x\le 4 \end{cases}$

Let $g(x)=-f(|x|)$. Find the area bounded by the graphs of f and g.

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Round 6 – Trig and Complex Numbers

1. ____________________

2. ____________________

3. ____________________

1. Let x be a real number such that $0 \le x \le 2\pi$. Compute the set of x-values for which the function $f(x) = \sin[x]$ takes on its maximum value. Note: $[x]$ stands for the greatest integer function.

2. $\overline{AB}$ is a diameter of circle O, $\overleftrightarrow{BD}$ is tangent to the circle at B and $\overline{AD}$ intersects the circle at E. C lies on $\overline{BD}$ between B and D such that $\overleftrightarrow{EC}$ is tangent to the circle at E. If $\overleftrightarrow{EC}$ bisects $\angle BED$, compute $\tan \angle BDA$.

3. The function $r = \dfrac{4}{2\cos\theta + \sin\theta}$ is in polar coordinates. Compute the smallest <u>positive</u> value of r that the function takes on.

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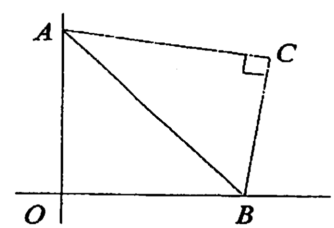
Team Round

Place answers on Team Round Answer Sheet

1. Let a and b be four-digit positive integers with $a < b$. Compute the number of ordered pairs (a,b) such that the sum $a+b$ is a four-digit number.

2. Circles P and Q are externally tangent at T. At time $t = 0$, points A and B are both at T. Then, at the same time, A and B start moving, A traveling counterclockwise on P and B traveling clockwise on Q. If A makes one revolution every 10 seconds and B makes one revolution every 16 seconds, compute the least possible positive value of t (in seconds) at which points P, Q, A, and B are collinear.

3. AOB is an isosceles right triangle and CAB is a 3-4-5 right triangle with the right angle at C and the short leg is $\overline{CB}$. Compute the slope of $\overline{OC}$. (Note: $\overleftrightarrow{OB}$ and $\overleftrightarrow{OA}$ are the x and y axes respectively)



4. This problem concerns positive integers. Compute all values of n such that the number of n-digit palindromes that are odd is 1,000,000 more than the number of n-digit palindromes that are even.

5. Let h be greater than 0 and not equal to 2. The graphs of $y = x^2$ and $y = (x-2)^2$ intersect at point A, the graphs of $y = (x-2)^2$ and $y = (x-h)^2$ intersect at B, and the graphs of $y = x^2$ and $y = (x-h)^2$ intersect at C. If $CA = CB$, compute the sum of all possible positive values of h.

6. ΔABC is isosceles. Compute the minimum value of $\dfrac{\sin A + \sin B + \sin C}{\sin A \cdot \sin B \cdot \sin C}$.

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STATE PLAYOFFS – 2014 – ANSWER SHEET

Round 1

1. 6
2. 507
3. 3

Round 2

1. 31
2. 20
3. $x \le -1$

Round 3

1. 24
2. $\frac{1}{13}$
3. $\frac{1+\sqrt{5}}{2}$

Round 4

1. 101
2. -3
3. $(-1,-1,-1), (3,3,3)$

Round 5

1. $\left(\frac{5}{3}, -2\right)$
2. 18
3. 18

Round 6

1. $2 \le x < 3$
2. 1
3. $\frac{4\sqrt{5}}{5}$

Team

1. 16,000,000
2. 40
3. $\frac{3}{4}$
4. 13, 14
5. $2 + 2\sqrt{2}$
6. 4

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Solutions: State Meet 2014

Round 1 Arithmetic and Number Theory:

1. Note that the numbers must be less than 6. If $a=5$, then $b^3+c^3=28$ and only 1 and 3 satisfy that equation. Thus, all six permutations of (5, 1, 3) satisfy the equation. If $a=4$, then $b^3+c^3=89$ and if b also equals 4, then $c^3=25$ and there are no solutions. If a is less than 4 there are no solutions. Answer: $\boxed{6}$.

2. Since a_9 has 9 factors it must be a perfect square and must be of the form p^8 or $p_1{}^2 \cdot p_2{}^2$. Since $a_6=117=3^2\cdot 13$, then $a_9=3^2\cdot 13^2$, making $a_8=\dfrac{3^2\cdot 13^2}{3} = \boxed{507}$.

3. Since ABCD < BCDA and ABCD + BCDA = 6897, then B < 6, A + B = 6, and A ≤ B. Let A = 1 and B = 5. Since A + D = 7, then D = 6, making C = 3, giving (1536, 5361). Reasoning in this fashion we obtain (1536, 5361), (2445, 4452), and (3354, 3543). Answer: $\boxed{3}$.

Round 2 Algebra I:

1. $a^3+b^3=(a+b)\left(a^2-ab+b^2\right)=126 \rightarrow a^2-ab+b^2=42$ since $a+b=3$. From $a+b=3$ we obtain $a^2+2ab+b^2=9$. Multiply $a^2-ab+b^2=42$ by 2 and add the result to $a^2+2ab+b^2=9$ to eliminate the ab term, resulting in $3a^2+3b^2=93$. Thus, $\boxed{a^2+b^2=31}$.

2. From $\dfrac{a+b}{b+c}=\dfrac{1}{2}$ we obtain $2a+2b=b+c$. Add a to both sides obtaining $3a+2b=a+b+c=40$.

From $\dfrac{a+b}{a+c}=\dfrac{1}{5}$ we obtain $5a+5b=a+c$. Here we add b to both sides to obtain

$5a+6b=a+b+c=40$. Thus we have the system $3a+2b=40$ and $5a+6b=40$. Multiply the first equation by 3 and subtract giving $4a=80$ so $\boxed{a=20}$.

3. First check the domain of the equation. The right side is defined for all x. On the left side we have $|x-1| \geq 1 \rightarrow x-1 \leq -1$ or $x-1 \geq 1 \rightarrow x \leq 0$ or $x \geq 2$.

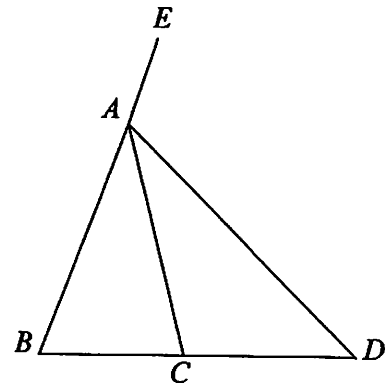
Square both sides to obtain $|x-1|-1=|x+1|+1$. Replace the absolute value expressions according to these three cases:

i) $x \geq 1 \rightarrow x-1-1=x+1+1 \rightarrow -2=2$. There are no solutions in this interval.
ii) $-1 < x < 1 \rightarrow 1-x-1=x+1+1 \rightarrow -2=2x \rightarrow x=-1$. No solutions in this interval.
iii) $x \leq -1 \rightarrow 1-x-1=-x-1+1 \rightarrow 0=0$. This is true for all x, giving $x \leq -1$ for the solution set.

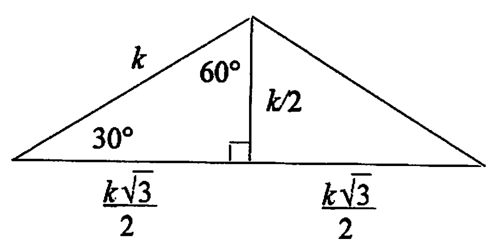
Since all the solutions in (iii) lie in the domain, the answer is $\boxed{x \leq -1}$.

Round 3 Geometry:

1. The general case is more interesting so let $m\angle B=\theta$, making $m\angle ACB=\theta$. By the Exterior Angle Theorem, $m\angle CAE=2\theta$. Draw the trisector closer to $\overline{AC}$. Then $m\angle CAD=\frac{1}{3}\cdot 2\theta=\frac{2}{3}\theta$. Since $m\angle DCA=180-\theta$, then $m\angle CDA=180-(180-\theta)-\frac{2\theta}{3}=\frac{\theta}{3}$. In this case $\theta=72$, so the answer is $\boxed{24}$. The trisector closer to $\overline{AE}$ does not meet side $\overline{BC}$ extended past C.



2. As the diagram shows, in a 30-120-30 triangle the ratio of the long side to the short side is $\sqrt{3}:1$. Thus, the ratio of the area of ACD to the area of ABC is 3:1. Likewise the ratio of the area of ADE to ACD is 3:1, so the ratio of ADE. to ABC is 9:1. Finally,



$$\frac{a(\Delta ABC)}{a(ABCDE)}=\frac{a(\Delta ABC)}{a(\Delta ABC)+a(\Delta ACD)+(\Delta ADE)}=\frac{a(\Delta ABC)}{a(\Delta ABC)+3a(\Delta ABC)+9a(\Delta ABC)}=\boxed{\frac{1}{13}}.$$

3. Let x = the side of the smaller square and y the side of the larger. Then $\frac{4x}{4y}=\frac{x^2}{y^2-x^2}\rightarrow\frac{1}{y}=\frac{x}{y^2-x^2}\rightarrow y^2-x^2=xy\rightarrow y^2-xy-x^2=0$. Dividing both sides by x^2 gives $\left(\frac{y}{x}\right)^2-\frac{y}{x}-1=0$. Solving: $\boxed{\frac{y}{x}=\frac{1+\sqrt{5}}{2}}$.

Round 4 Algebra II

1. Let the numbers be $a-40$, $a-20$, a, $a+20$, and $a+40$. As long as the sum of the two smallest exceeds the largest, then any three lengths will form a triangle. From $(a-40)+(a-20)>(a+40)$ we obtain $a>100$. Thus, $\boxed{a=101}$

2. Subtracting the first from the second gives $y-x=1 \to y=x+1$. Subtracting the first from the third gives $z-x=2 \to z=x+2$. Replacing y and z by their equivalents in x in the first equation gives $x+x(x+1)(x+2)=7 \to x^3+3x^2+3x-7=0$. By the relationship between the roots and coefficients of a polynomial we know that the sum of the roots equals –3. Answer: $\boxed{-3}$.

   Alternately, we can find the roots. The rational root theorem says that ±1, ±7 are the only possible rational roots. We find that 1 works, giving the following factorization of the polynomial: $(x-1)(x^2+4x+7)=0$. Using the quadratic formula we obtain the other two roots as $-2 \pm i\sqrt{3}$ and the sum is again –3.

   The solutions are $(x,y,z)=(1,2,3)$, $\left(-2+i\sqrt{3},-1+i\sqrt{3},i\sqrt{3}\right)$, $\left(-2-i\sqrt{3},-1-i\sqrt{3},-i\sqrt{3}\right)$.

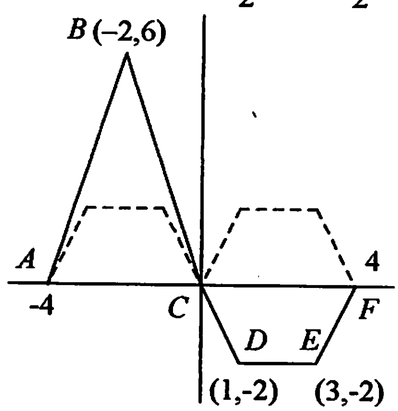
3. We have $ab=a+b+3, ac=a+c+3, bc=b+c+3$. Subtracting the first two given equations gives $(b-c)=b-c$. If $b=c$, then a is undetermined. From the other two pairs of equations, we get $a=b$ and $a=c$. Therefore $a=b=c$. Then $a^2-2a-3=0$ giving $a=$ 1 or $a=\ -3$. $\boxed{(-1,-1,-1);\ (3,3,3)}$

Round 5 Analytic Geometry:

1. $(x-3)^2+(y+2)^2=25$. For y- intercepts $(0-3)^2+(y+2)^2=25 \to y=2$ or $y=-6$. Using, (0,2) and the center $(3,-2)$, $x=\frac{1}{4p}(y+2)^2+3 \to 0=\frac{1}{4p}(2+2)^2+3 \to -3=\frac{1}{4p}\cdot 16$ $\to -3=\frac{4}{p} \to p=-\frac{4}{3}$. The focal point would be at $\left(3-\frac{4}{3},-2\right)=\left(\frac{5}{3},-2\right)$.

2. Dividing gives $y=\frac{x^2+2x-6}{x-2}=x+4+\frac{2}{x-2}$. The vertical asymptote is $x=2$, and the slant asymptote, $y=x+4$, intersects the x-axis at $x=-4$. The region in question is a triangle with a base equal to $2--4=6$ and the vertical asymptote intersects the slant asymptote at (2, 6) so the height is 6. The area is $\frac{1}{2}\cdot 6\cdot 6=\boxed{18}$.

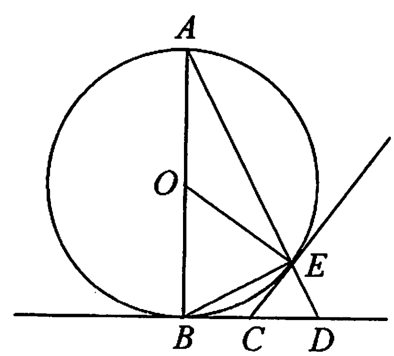
3. The graph of f consists of triangle ABC with $A=(-4,0)$, $B=(-2,6)$, and $C=(0,0)$ along with trapezoid $CDEF$ where $D=(1,2)$, $E=(3,-2)$, and $F=(4,0)$. The graph of g consists of $CDEF$ reflected across the y-axis and then across the x-axis plus the graph of $CDEF$ reflected across the x-axis. Upon inspection it can be seen that the area bounded by the two functions is equal to the area bounded by the graph of f and the x-axis. This is $\frac{1}{2}\cdot 4\cdot 6+\frac{1}{2}\cdot 2\cdot(4+2)=\boxed{18}$.



Round 6 Trigonometry and Complex Numbers:

1. Since $y = \sin x$ takes on its largest value of 1 at $x = \frac{\pi}{2} \approx 1.57$ and because $[x]$ takes on the values 0, 1, 2, 3, 4, 5, 6 for $0 \le x \le 2\pi$, $f(x) = \sin[x]$ will take on its largest value when $[x] = 2$ since that is closer to $\frac{\pi}{2}$ than $[x] = 1$ is. Thus, the answer is $\boxed{2 \le x < 3}$.

2. Consider the diagram below. Since ΔABE is inscribed in a semicircle, $m\angle BEA = 90$. Thus, $m\angle BED = 90$, and since $\overline{EC}$ bisects $\angle BED$, then $m\angle BEC = 45$. Since the sides of angle BEC are a secant and a tangent, $m\widehat{BE} = 90$. Since $\angle A = \frac{1}{2} m\widehat{BE}$, then $m\angle A = 45$, making $m\angle BDA = 45$, so $\tan\angle BDA = 1$ $\boxed{1}$.



3. Convert $r=\dfrac{4}{2\cos\theta+\sin\theta}$ to rectangular form: $r=\dfrac{4}{\frac{2x}{r}+\frac{y}{r}}\rightarrow r=\dfrac{4r}{2x+y}\rightarrow$

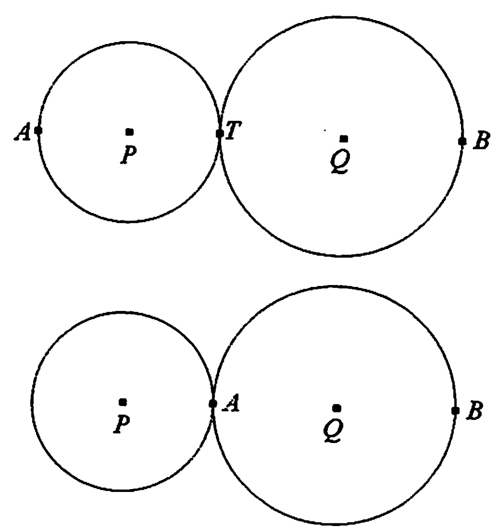
$1=\dfrac{4}{2x+y}\rightarrow 2x+y=4$. Use the distance formula from a point to a line using (0, 0) and obtain

$d=\dfrac{|2\cdot 0+1\cdot 0-4|}{\sqrt{2^2+1^2}}=\dfrac{4}{\sqrt{5}}=\boxed{\dfrac{4\sqrt{5}}{5}}$.

Team :

1. If $b=8999$, then $a=1000$. If $b=8998$, $a=1000$ or 1001. If $b=8997$, then a could equal 1000, 1001, or 1002. Note that as b decreases by 1, the number of values of a increases by 1. This continues to $b=5000$ where a could be any integer from 1000 to 4999 inclusive, a total of 4000 values. This gives 1 + 2 + . . . + 4000 possible ordered pairs. As b continues to decrease below 5000, it turns out that the number of values of a also decreases. If $b=4999$, a can take on values from 1000 to 4998 inclusive, if $b=4998$, a can take on values from 1000 to 4997 inclusive, and so on down to $b=1002$ where a = 1000 or 1001, and finally if $b=1001$ then a can only be 1000. This gives 3999 + 3998 + . . . + 2 + 1 possible ordered pairs. The final total is
$\dfrac{1+4000}{2}\cdot 4000+\dfrac{1+3999}{2}\cdot 3999 = 2000(4001+3999) = \boxed{16{,}000{,}000}$.

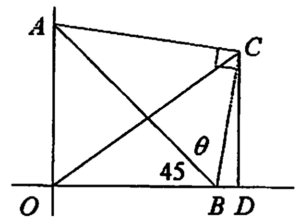
2. A makes 1/10 revolution per second and B makes 1/16 revolution per second. When points P, Q, A, and B are collinear, there are 3 possibilities: i) A and B are both back at T, ii) A and B are outside P and Q, or iii) one of A and B is at T and the other is outside. Cases (ii) and (iii) are shown in the diagram. In (i) A and B have both made an integer number of revolutions; the difference in the number of revolutions is also an integer. In (ii) each has made half a revolution more than an integral number but the difference in the number of revolutions is still an integer. In (iii) one has made an integral number while the other has made an integer and a half revolution more so the difference in the number of revolutions involves half a revolution. Since we must allow for any case, we have $\dfrac{1}{10}t-\dfrac{1}{16}t=\dfrac{n}{2}$. This



simplifies to $n=\frac{3t}{40}$ so $\boxed{t=40}$ is the least value of t at which the points are collinear. At 40 seconds A has made 4 complete revolutions and B has made 2.5.

Note: Either A and B are traveling at different rates around congruent circles or they are travelling at the same rate around circles of different sizes. In our diagrams we assume the latter.

3. Drop a perpendicular from $C(a,b)$ meeting the x-axis at D. Let $m\angle ABC=\theta$ and $m\angle CBD=\alpha$. Without loss of generality, set $AB=1$, making $BC=\frac{3}{5}$ and $AC=\frac{4}{5}$. Since $\alpha=180-(45+\theta)=$



$135-\theta$ and $b=CD=\frac{3}{5}\sin\alpha$, then $b=\frac{3}{5}\sin(135-\theta)=\frac{3}{5}(\sin 135\cos\theta-\cos 135\sin\theta)$ = $\frac{3}{5}\left(\frac{\sqrt{2}}{2}\cdot\frac{3}{5} - \frac{-\sqrt{2}}{2}\cdot\frac{4}{5}\right) = \frac{21\sqrt{2}}{50}$. Also, $a=OD=OB+BD=\frac{\sqrt{2}}{2}+\frac{3}{5}\cos\alpha$ = $\frac{\sqrt{2}}{2}+\frac{3}{5}\cos(135-\theta)=\frac{\sqrt{2}}{2}+\frac{3}{5}(\cos 135\cos\theta+\sin 135\sin\theta) = \frac{\sqrt{2}}{2}+\frac{3}{5}\left(\frac{-\sqrt{2}}{2}\cdot\frac{3}{5}+\frac{\sqrt{2}}{2}\cdot\frac{4}{5}\right) = \frac{\sqrt{2}}{2}+\frac{3\sqrt{2}}{50}=\frac{28\sqrt{2}}{50}$. The slope of $\overline{CD}=\frac{b}{a}=\frac{\frac{21\sqrt{2}}{50}}{\frac{28\sqrt{2}}{50}} = \boxed{\frac{3}{4}}$.

Alternate Solution:

Let $OB = OA = 5\sqrt{2}$, $BC = 6$, $CA = 8$. Then $AB = 10$. Then with $\tan\angle ABC = \frac{4}{3}$, we have $\tan\angle OBC = \frac{1+4/3}{1-4/3} = -7$. Drop a perpendicular from C intersecting $\overline{OB}$ extended at D. Then. $\tan\angle CBD = 7$, $\cos\angle CBD = \frac{1}{5\sqrt{2}}$ and $\sin\angle CBD = \frac{7}{5\sqrt{2}}$. Then $CD = \frac{42}{5\sqrt{2}}$ and $BD = \frac{6}{5\sqrt{2}}$ making $OD = \frac{56}{5\sqrt{2}}$. Then the slope of $\overline{OC} = \frac{42}{56} = \frac{3}{4}$.

4. If n is odd, the number of odd palindromes is 5(10)(10) . . . (10) where there are $\frac{n+1}{2}$ terms in the product and $\frac{n+1}{2}-1=\frac{n-1}{2}$ tens. The product is $5\cdot 10^{\frac{n-1}{2}}$. The number of even palindromes is 4(10)(10)...(10) where there are $\frac{n+1}{2}$ terms in the product. The product is $4\cdot 10^{\frac{n-1}{2}}$ giving a difference of $10^{\frac{n-1}{2}}$. Setting this equal to 10^6 gives $\frac{n-1}{2}=6 \rightarrow n=13$. If n is even, the number

of odd palindromes is 5(10)(10) . . . (10) where there are $\frac{n}{2}$ terms in the product, giving a product of $5\cdot 10^{\frac{n}{2}-1}$. The number of even palindromes would be $4\cdot 10^{\frac{n}{2}-1}$, giving a difference of $10^{\frac{n}{2}-1}$. Setting $\frac{n}{2}-1=6$ gives $n=14$. Thus, $\boxed{n=13 \text{ or } 14}$.

5. There are several cases to consider:

1) For A, $x^2=x^2-4x+4\to x=1$, giving $A=(1,1)$.

2) For B, $(x-2)^2=(x-h)^2\to x-2=h-x\to x=\frac{h+2}{2}=\frac{h}{2}+1$. Then $B=\left(\frac{h}{2}+1, \left(\frac{2-h}{2}\right)^2\right)$.

3) For C, $x^2=(x-h)^2\to 2xh=h^2\to x=\frac{h}{2}$. Then $C=\left(\frac{h}{2},\frac{h^2}{4}\right)$.

$AC=CA\to\sqrt{\left(\frac{h}{2}-1\right)^2+\left(\frac{h^2}{4}-1\right)^2}=\sqrt{\left(\frac{h}{2}+1-\frac{h}{2}\right)^2+\left(\frac{h^2}{4}-\left(\frac{4-4h+h^2}{4}\right)\right)^2}$. Squaring and simplifying gives $\frac{h^2}{4}-h+1+\frac{h^4}{16}-\frac{h^2}{2}+1=1+\left(1-2h+h^2\right)\to h^4-20h^2+16h=0$. Since $h>0$, we consider the cubic $h^3-20h+16=0$. The Rational Root Theorem suggests that 4 could be a solution and we find it is, giving $(h-4)\left(h^2+4h-4\right)=0$. The quadratic's solutions are $-2\pm2\sqrt{2}$. The positive values for h are 4 and $-2+2\sqrt{2}$. Their sum is $\boxed{2+2\sqrt{2}}$.

6. Let $m\angle A=m\angle C$. We have $\frac{\sin A+\sin B+\sin C}{\sin A\cdot\sin B\cdot\sin C}=\frac{2\sin A+\sin B}{\sin^2 A\cdot\sin B}=\frac{2\sin A+\sin(180-2A)}{\sin^2 A\cdot\sin(180-2A)}=$ $\frac{2\sin A+\sin 2A}{\sin^2 A\cdot\sin 2A}=\frac{2\sin A+2\sin A\cos A}{2\sin^3 A\cos A}=\frac{1+\cos A}{\sin^2 A\cos A}$. Multiplying top and bottom by $1-\cos A$ gives $\frac{1-\cos^2 A}{\sin^2 A\cdot\cos A\cdot(1-\cos A)}=\frac{1}{\cos A\cdot(1-\cos A)}$. The denominator is a quadratic and has a maximum value of 1/4, so the fraction has a minimum value of $\boxed{4}$.

Note: Since, in the expression to be minimized, A, B, and C can be interchanged without changing the expression, we may assume B is the vertex angle and A and C are the base angles without any loss of generality.