CANTON-L

LEAGUES

MAML

STATE INVITATIONAL MATH LEAGUE COMPETITION

April 4, 2008

MASSACHUSETTS ASSOCIATION OF MATHEMATICS LEAGUES

STATE PLAYOFFS – 2008

Round 1 **Arithmetic and Number Theory**

1. ______________________

2. ______________________

3. ______________________

1. Determine the largest five-digit number that is divisible by both 4 and 11.

2. Super Bowls are numbered using Roman Numerals starting with I, II, III, IV, V, VI, VII, and VIII. It took 8 digits to number the first four Super Bowls. What is the least possible number of digits that could be used to number the first forty-five Super Bowls?
(Roman Numerals: I -1, V – 5, X – 10, L – 50, C – 100)

3. Compute the sum of all six-digit palindromes in base 3 that use only 1's and/or 2's. Write the sum in base 10.

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Round 2 **Algebra 1**

1. ______________________

2. ______________________

3. ______________________

1. To purchase a candy bar, Sam paid one dollar and received change in dimes and pennies. After he lost half the dimes and half the pennies, he had twice as many dimes as the number of pennies that he'd received originally. What did the candy bar cost?

2. Given integers n and m with $n > m > 0$, determine the number of ordered pairs (n, m) such that $\left(2^n - 2^{n-1}\right)\left(2^m - 2^{m-1}\right) = 2^{2008}$.

3. Determine all ordered pairs (x, y) which satisfy the system:

$$\sqrt{x} + \sqrt{y} = 2$$

$$x + y = \frac{5}{2}$$

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Round 3 – Geometry

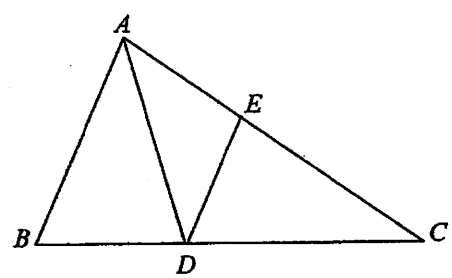
1. ____________________

2. ____________________

3. ____________________

1. In ΔABC, $AB = AC = 50$ and $BC = 60$. E is on $\overline{AC}$ so that $\overline{BE}$ is perpendicular to $\overline{AC}$. Find the area of triangle BEA.

2. In ΔABC, $\overline{AD}$ bisects $\angle BAC$, $\overline{DE} \parallel \overline{AB}$, $AE = 2$, and $EC = 5$. Find AB.



3. $ABCD$ is a regular tetrahedron of edge 6. A circle is inscribed in face BCD and a regular hexagon is inscribed in the circle. Determine the sum of the squares of the six distances from A to the vertices of the hexagon.

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Round 4 – Algebra 2

1. ____________________

2. ____________________

3. _(___,___)_(___,___)

1. Solve for all real x: $(x+1)^5 - (x-1)^5 = 352$.

2. Let $[x]$ = the greatest integer less than or equal to x. Solve for all real x:

   $[|x|] - |[x]| = -1$.

3. Real roots of $x = \dfrac{2}{\sqrt{1+\dfrac{1}{\sqrt{1+x}}}}$ lie between two pair of consecutive integers. What are the two pair?

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Round 5 – Analytic Geometry

1. ___(______,______)___

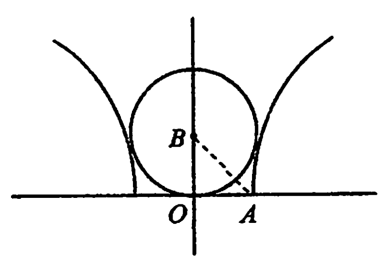
2. ____________________

3. ____________________

1. Point A lies in the first quadrant. A is reflected across the y-axis to B and then B is reflected across the x-axis to C. If the area of triangle ABC is 216 and the slope of $\overline{AC}$ equals 3, determine the coordinates of A.

2. For $a, b > 0$, the vertices of a right triangle are $(a, 0)$, $(0, b)$, and $(6, 4)$. If $a + b = 12$, determine the square of the hypotenuse.

3. Circle B is tangent to the x-axis and to both branches of the hyperbola $x^2 - ay^2 = m^2$ as shown in the diagram. Point A is an x-intercept of the hyperbola and O is the origin. If $OA : OB : AB = 3 : 4 : 5$, find the value of a.



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Round 6 – Trig and Complex Numbers

1. ____________________

2. ____________________

3. ____________________

1. In $\Delta ABC, AB = AC = 9$ and $m\angle B = 70°$. For θ in degrees, $BC = k\sin\theta$. Find the ordered pair (k, θ), where θ is acute.

2. For those values of x in $[0, 2\pi]$ for which the equation is defined, solve for x: $4(\sin x - \cos x) = \sec x - \csc x$.

3. For $0 \le \theta \le 2\pi$, determine the sum of the θ-coordinates of all points of intersection of the polar curves $r = \sin 2\theta$ and $r = \frac{1}{2}$.

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Team Round

1. ____________________   4. ________________

2. ____________________   5. ________________

3. ____(______,______)____   6. ________________

1. In adding up the numbers from 1 to n, Ralph mistakenly added a certain number three times instead of once and obtained a sum of 2009. What are the possible values for that number?

2. Slim has nine distinguishable objects. He wants to distribute them into three distinguishable containers, a bag, a box, and a big bottle. In how many ways can Slim distribute the objects such that the product of the number of objects in any two containers is odd?

3. Consider a frustum of a right circular cone in which the radii of the two bases are x and $3x$ respectively and the slant height is y. If $x, y \varepsilon \{1,2,3,\ldots,2007,2008\}$, determine the number of ordered pairs (x,y) such that the lateral surface area of the frustum equals the sum of the areas of its bases.

4. Let T be the set of all two-digit positive integers that have at least one 3 and no 2's and let S be the set of all two-digit whole numbers that have at least one 2 and no 3's. Let $t_1, t_2, \ldots, t_n$ be a permutation of the elements of T and $s_1, s_2, \ldots, s_n$ be a permutation of the elements of S. Determine the value of $(t_1 - s_1) + (t_2 - s_2) + \cdots + (t_n - s_n)$.

5. Determine the number of ordered pairs (x, y) of positive integers such that

$$\tan^{-1}\left(\frac{2}{x}\right) + \tan^{-1}\left(\frac{5}{y}\right) = \frac{\pi}{4}.$$

6. If $3 \le a \le 9$, $4 \le b \le 20$, and $a \ne b$, determine the smallest positive solution to $\sqrt{x-a} = \sqrt{x-b} + \sqrt{a}$.

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Answer Sheet

Round 1

1. 99968
2. 161
3. 4368

Round 2

1. \$0.18
2. 1004
3. $\left(\frac{9}{4}, \frac{1}{4}\right)$ and $\left(\frac{1}{4}, \frac{9}{4}\right)$

Round 3

1. 336
2. $\frac{14}{5}$
3. 162

Round 4

1. $\pm\sqrt{5}$
2. all non-integer negative numbers
3. (1, 2), (2, 3)

Round 5

1. (6, 18)
2. 104
3. $\frac{7}{9}$

Round 6

1. (18, 20)
2. $-\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{\pi}{4}, \frac{5\pi}{4}$
3. 8π

Team

1. 28, 59
2. 4920
3. 401
4. 98
5. 6
6. $\frac{13}{3}$

Solutions: State Meet 2008

Round 1 Arithmetic and Number Theory:

1. Divide 99,999 by 44 and obtain 2272 with a remainder of 31. Subtract 31 from 99,999 to obtain the answer: 99,968.

2. Numbers 1 – 10 are: I, II, III, IV, V, VI, VII, VIII, IX, X. They use 21 digits. For 11 – 20 an X is added to the beginning of each number, giving 31 digits. For 21 – 30, an X is added to the beginning, giving 41 digits. For 31 – 40 we need only 51 – 2 = 49 digits. 41 = XLI, 42 = XLII, 43 = XLIII, 44 = XLIV, and 45 = XLV, using 19 digits. The total is 21 + 31 + 41 + 49 + 19 = 161.

3. The palindrome will be of the form ABCCBA. We need only concern ourselves with the first three places since the other three are determined. We have 2 choices for each place making for $2^3 = 8$ different palindromes. Each place will be filled by a 1 in four of the palindromes and by a 2 in the other four, making a sum of 12 for each place. Thus, the sum of all the numbers is

$$12\left(3^5 + 3^4 + 3^3 + 3^2 + 3^1 + 1\right) = 12 \cdot \frac{1-3^6}{1-3} = 4368.$$

Round 2 Algebra I:

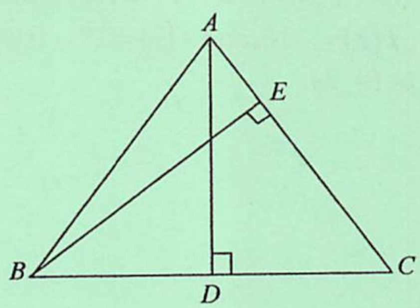
1. Let x = number of dimes and y = number of pennies received in change. Then he received a total of $10x + y$; after he lost half, he had $10 \cdot \frac{x}{2} + \frac{y}{2}$ where $\frac{x}{2} = 2y \to x = 4y$. Thus, he received $10(4y) + y = 41y$ in change. Note that y can't be 1 since we no longer have pennies. Thus, $y = 2$ and he received 82 cents in change. The candy bar cost 18 cents.

2. $\left(2^n - 2^{n-1}\right)\left(2^m - 2^{m-1}\right) = 2^{n-1}(2-1)2^{m-1}(2-1) = 2^{n+m-2} = 2^{2008}$. Thus, $n + m = 2010$ and since $n > m > 0$, $1006 \le n \le 2009$ and for each n we have $m = 2010 - n$, making $1 \le m \le 1004$. Thus, there are 1004 pairs.

3. Squaring the first equation gives $x + 2\sqrt{xy} + y = 4 \to xy = \frac{9}{16} \to y = \frac{9}{16x}$. Thus, $x + \frac{9}{16x} = \frac{5}{2} \to 16x^2 - 40x + 9 = 0$ giving $(4x-1)(4x-9) = 0$. Ans: $\left(\frac{1}{4}, \frac{9}{4}\right)$ and $\left(\frac{9}{4}, \frac{1}{4}\right)$.

Round 3 Geometry:

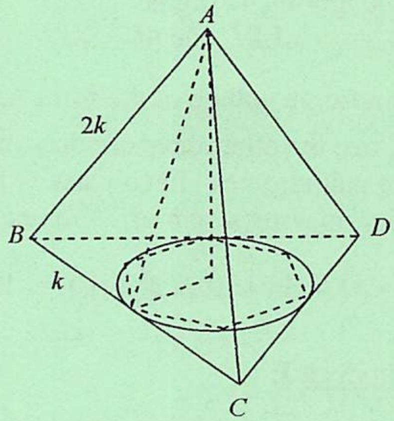
1. Draw altitude $\overline{AD}$, making the 30-40-50 triangle ADC.
Since $\Delta BEC \sim \Delta ADC$, $\frac{BC}{BE} = \frac{60}{BE} = \frac{5}{4}$, so $BE = 48$.
Similarly, $EC = 36$, making $AE = 14$. Area BEA = $(.5)(14)(48) = 336$.



2. Since $\overline{AB} \parallel \overline{DE}$ then $\frac{BD}{DC} = \frac{2}{5}$. By the Triangle Angle Bisector Theorem $\frac{AB}{AC} = \frac{BD}{DC}$, so $\frac{AB}{7} = \frac{2}{5}$, making $AB = \boxed{\frac{14}{5}}$.

Alternate Solution: alternate interior angles makes triangle AED isosceles, so $ED = 2$. Use similar triangles to obtain 14/5.

3. The general case is more interesting. Let the edge of the tetrahedron equal $2k$. The right triangle formed by the altitude of the tetrahedron and the distance from the center of the inscribed circle to a vertex of the inscribed regular hexagon has a hypotenuse equal to the altitude of a face and that is equal to $k\sqrt{3}$. Thus, the sum of the squares of the six distances to the vertices of the inscribed hexagon equals $6\left(k\sqrt{3}\right)^2 = 18k^2$. Since the edge has a length of 6, then $k = 3$ making $18k^2 = 162$.



Round 4 Algebra II:

1. $(x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1) - (x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1) = 352 \rightarrow$ $10x^4 + 20x^2 + 2 = 352 \rightarrow x^4 + 2x^2 - 35 = 0 \rightarrow (x^2 + 7)(x^2 - 5) = 0$. Thus, $x = \pm\sqrt{5}$.

2. If $x > 0$, then $[x] - [x] = -1$ and there are no solutions. If x is a negative integer, then the left side equals 0, so no solutions. If x is negative and not an integer, let $x = n + h$ where n is a negative integer and $0 < h < 1$. Then $\left[\left|n + h\right|\right] = \left|n - 1\right|$ and $\left|\left[n + h\right]\right| = \left|n\right|$. The difference is -1. Thus, the solution set consists of all negative numbers that are not integers.

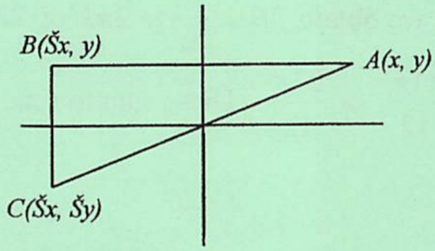
3. $\frac{2}{x} = \sqrt{1 + \frac{1}{\sqrt{1+x}}} \rightarrow \frac{4}{x^2} - 1 = \frac{1}{\sqrt{x+1}} \rightarrow x^4 = \left(4 - x^2\right)^2(x+1)$. This gives $x^5 - 8x^3 - 8x^2 + 16x + 16 = 0$. Letting this expression equal $f(x)$ we have, $f(0) = 16$, $f(1) = 17$, $f(2) = -16$. and $f(3) = 19$. By the Intermediate Value Theorem, we could have a solution in (1, 2), or (2, 3).

Round 5 Analytic Geometry:

1. $216 = \frac{1}{2} \cdot BC \cdot AB = \frac{1}{2} \cdot 2y \cdot 2x \rightarrow$

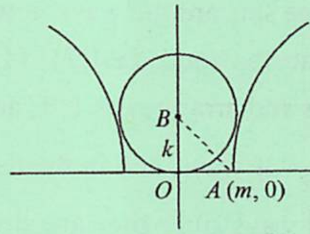
$xy = 108$. $m\overline{AC} = 3 \rightarrow \frac{2y}{2x} = 3 \rightarrow y = 3x$.

So, $3x^2 = 108 \rightarrow x = 6$.



2. Given $A = (a,0), B = (0,b)$, and $C = (6,4)$, set $AB^2 = AC^2 + BC^2$, obtaining $a^2 + b^2 = (a-6)^2 + 4^2 + 6^2 + (b-4)^2 \rightarrow 26 = 3a + 2b$. Using $a + b = 12$ we obtain $a = 2$ and $b = 10$. Thus, $a^2 + b^2 = 104$.

3. Let $B = (0,k)$ and $A = (m,0)$. The equation of the circle is $x^2 + (y-k)^2 = k^2$. Subtracting $x^2 - ay^2 = m^2$, we obtain $(y-k)^2 + ay^2 = k^2 - m^2$, giving $(a+1)y^2 - 2ky + m^2 = 0$. This has one solution so $(2k)^2 - 4(a+1)m^2 = 0 \rightarrow k = m\sqrt{a+1}$. Since $\frac{OB}{OA} = \frac{k}{m} = \frac{4}{3}$, then $\sqrt{a+1} = \frac{4}{3} \rightarrow a = \frac{7}{9}$.



Round 6 Trigonometry and Complex Numbers:

1. By the Law of Cosines, $BC^2 = 2 \cdot 9^2 - 2 \cdot 9^2 \cos 40° = 162(1 - \cos 40°) = 324\left(\frac{1-\cos 40°}{2}\right) = 324\left(\sin^2 20°\right)$. Thus, $BC = 18(\sin 20°)$. Answer: $(18, 20°)$.

2. $4(\sin x - \cos x) = \sec x - \csc x = \frac{1}{\cos x} - \frac{1}{\sin x} = \frac{\sin x - \cos x}{\cos x \cdot \sin x}$. We have $\sin x - \cos x = 0$, giving $x = \frac{\pi}{4}$ or $\frac{3\pi}{4}$, or $4 = \frac{1}{\sin x \cdot \cos x}$, giving $2\sin x \cos x = \frac{1}{2} \rightarrow$ $\sin 2x = \frac{1}{2} \rightarrow 2x = \frac{\pi}{6} + 2\pi k$ or $\frac{5\pi}{6} + 2\pi k$. Hence $x = \frac{\pi}{12} + \pi k$ or $\frac{5\pi}{12} + \pi k$. The solutions are $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{\pi}{4}, \frac{3\pi}{4}$.

3. Solving $r = \sin 2\theta$ and $r = \frac{1}{2}$ simultaneously we obtain $2\theta = \frac{\pi}{6} + 2\pi k$ or $2\theta = \frac{5\pi}{6} + 2\pi k$, giving $\theta = \frac{\pi}{12}, \frac{13\pi}{12}, \frac{5\pi}{12}$, or $\frac{17\pi}{12}$. These sum to 3π. But $r = \sin 2\theta$ has 4 petals so we have only half the intersections. The other four can be determined by symmetry or by realizing that since descriptions of points in polar coordinates are not unique, neither are equations describing those

points. So consider another way to graph the circle, i.e., $r = -\frac{1}{2}$. Solving $r = \sin 2\theta$ and $r = -\frac{1}{2}$ simultaneously, we obtain $2\theta = \frac{7\pi}{6} + 2\pi k$ or $2\theta = \frac{11\pi}{6} + 2\pi k$, giving $\theta = \frac{7\pi}{12}, \frac{19\pi}{12}, \frac{11\pi}{12}$, or $\frac{23\pi}{12}$. These sum to 5π, giving a final answer of 8π.

Team:

1. Let k be the integer added twice. Then $\frac{n(n+1)}{2} + 2k = 2009 \rightarrow n(n+1) + 4k = 4018$. We note that $63(64) = 4032$ so $n \leq 62$. If $n = 62$, $4k = 4018 - 62(63) = 112$, making $k = 28$. If $n = 61$, $4k = 61(62) = 236$, making $k = 59$. Clearly the values of n less than 61, k will be larger than n. So the answers are $k = 28$ or 59.

2. These three sets are the ways in which we can distribute the objects so that the product of any two of the numbers is odd: {1,1,7}, {1,3,5}, and {3,3,3}. In the first set, we can pick the 7 objects in ${}_9C_7$ ways and arrange the 1, 1, and 7 in 3! ways. In the second set, we can choose and arrange in ${}_9C_5 \cdot {}_4C_3 \cdot 3!$ ways, and in the third set we can choose in ${}_9C_3 \cdot {}_6C_3$ ways. We cannot arrange these in 3! ways since they are already completely arranged. The total is $3!\left({}_9C_7 + {}_9C_5 \cdot {}_4C_3\right) + {}_9C_3 \cdot {}_6C_3 = 6(36 + 126(4)) + 84(20)) = 6(540) + 1680 = 4920.$

3. Completing the cone we have two triangles with a scale factor of 1:3. Thus, the slant height of the small cone is $\frac{y}{2}$ and the slant height of the large cone is $\frac{3y}{2}$. The lateral surface area of the frustum equals $\frac{1}{2} \cdot \pi \cdot 3x \cdot \frac{3y}{2} - \frac{1}{2} \cdot \pi \cdot x \cdot \frac{y}{2} = \frac{9\pi xy}{4} - \frac{\pi xy}{4} = 2\pi xy$. The sum of the areas of the bases equals $\pi x^2 + \pi(3x)^2 = 10\pi x^2$. From $10\pi x^2 = 2\pi xy$ we obtain $y = 5x$. Thus the ordered pairs are {(1, 5), (2, 10), . . . (401, 2005)}, giving a total of 401 ordered pairs.

4. The results should all be the same, namely $\left(t_1 + \mathrm{K} + t_n\right) - \left(s_1 + \mathrm{K} + s_n\right)$. We have

$$(13 + 23 + 30 + \ldots + 39 + 43 + \ldots + 93 - 23 - 32) - (12 + 20 + \ldots + 29 + 32 + \ldots + 92 - 23 - 32)$$

$$= 1 + 23 + \frac{(30 + 39)10}{2} + 6(1) - \frac{(20 + 29)10}{2} - 32 = 7 - 9 + 345 - 245 = 98.$$

5. Take the tangent of both sides: $\tan\left(\tan^{-1}\frac{2}{x} + \tan^{-1}\frac{5}{y}\right) = \tan\frac{\pi}{4} \rightarrow \frac{\frac{2}{x} + \frac{5}{y}}{1 - \frac{10}{xy}} = 1$. Solving for y gives $y = \frac{5x + 10}{x - 2} = 5 + \frac{20}{x - 2}$. Since 20 has 6 factors, there are 6 ordered pairs (x, y) that will generate solutions to the problem.

6. $\sqrt{x-a}-\sqrt{a}=\sqrt{x-b} \rightarrow x-a-2\sqrt{a}\sqrt{x-a}+a=x-b \rightarrow b=2\sqrt{a}\sqrt{x-a} \rightarrow$

$\frac{b^2}{4}=a(x-a) \rightarrow x=a+\frac{b^2}{4a}$. By the AM-GM, $\frac{a+\frac{b^2}{4a}}{2} \geq \sqrt{a \cdot \frac{b^2}{4a}} \rightarrow a+\frac{b^2}{4a} \geq b$. Thus, choose b as small as possible. Letting $b = 4$ makes $a+\frac{b^2}{4a}=a+\frac{16}{4a}=a+\frac{4}{a}$. We want to minimize $a+\frac{4}{a}$ over the interval [3, 9]. Since $a+\frac{4}{a} \geq 2\sqrt{a \cdot \frac{4}{a}}$, the minimum of $a+\frac{4}{a}$ is 4 and it occurs at $a=2$. The function $y=x+\frac{4}{x}$ is increasing after $x=2$ so we minimize $a+\frac{4}{a}$ over the interval [3, 9] by choosing the value closest to 2, namely 3. The minimum is therefore $3+\frac{4}{3}=$ $4\frac{1}{3}=\frac{13}{3}$.

2008

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1. Let k be the integer added twice. Then $\frac{n(n+1)}{2} + 2k = 2009 \to n(n+1) + 4k = 4018$. We note that $63(64) = 4032$ so $n \le 62$. If $n = 62$, $4k = 4018 - 62(63) = 112$, making $k = 28$. If $n = 61$, $4k = 61(62) = 236$, making $k = 59$. Clearly the values of n less than 61, k will be larger than n. So the answers are $k = 28$ or 59.

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4. The results should all be the same, namely $(t_1 + \text{K} + t_n) - (s_1 + \text{K} + s_n)$. We have

$$(13 + 23 + 30 + \ldots + 39 + 43 + \ldots + 93 - 23 - 32) - (12 + 20 + \ldots + 29 + 32 + \ldots + 92 - 23 - 32)$$

$$= 1 + 23 + \frac{(30+39)10}{2} + 6(1) - \frac{(20+29)10}{2} - 32 = 7 - 9 + 345 - 245 = 98.$$

5. Take the tangent of both sides: $\tan\left(\tan^{-1}\frac{2}{x} + \tan^{-1}\frac{5}{y}\right) = \tan\frac{\pi}{4} \to \frac{\frac{2}{x} + \frac{5}{y}}{1 - \frac{10}{xy}} = 1$. Solving for y gives $y = \frac{5x+10}{x-2} = 5 + \frac{20}{x-2}$. Since 20 has 6 factors, there are 6 ordered pairs (x,y) that will generate solutions to the problem.

6. $\sqrt{x-a}-\sqrt{a}=\sqrt{x-b} \rightarrow x-a-2\sqrt{a}\sqrt{x-a}+a=x-b \rightarrow b=2\sqrt{a}\sqrt{x-a} \rightarrow$

$\frac{b^2}{4}=a(x-a) \rightarrow x=a+\frac{b^2}{4a}$. By the AM-GM, $\frac{a+\frac{b^2}{4a}}{2} \geq \sqrt{a\cdot\frac{b^2}{4a}} \rightarrow a+\frac{b^2}{4a} \geq b$. Thus, choose b as small as possible. Letting $b=4$ makes $a+\frac{b^2}{4a}=a+\frac{16}{4a}=a+\frac{4}{a}$. We want to minimize $a+\frac{4}{a}$ over the interval [3, 9]. Since $a+\frac{4}{a} \geq 2\sqrt{a\cdot\frac{4}{a}}$, the minimum of $a+\frac{4}{a}$ is 4 and it occurs at $a=2$. The function $y=x+\frac{4}{x}$ is increasing after $x=2$ so we minimize $a+\frac{4}{a}$ over the interval [3, 9] by choosing the value closest to 2, namely 3. The minimum is therefore $3+\frac{4}{3}$ = $4\frac{1}{3}=\frac{13}{3}$.