57th ANNUAL

MASSACHUSETTS MATHEMATICS OLYMPIAD

2020 - 2021

A High School Competition Conducted by

THE MASSACHUSETTS ASSOCIATION
OF MATHEMATICS LEAGUES(MAML)

And Sponsored by

THE ACTUARIES' CLUB OF BOSTON

## FIRST-LEVEL EXAMINATION

Thursday, November 5, 2020

INSTRUCTIONS

1. **Proctoring: Since the examination is being given online, teachers will not be expected to proctor the exam. However, each student will be asked to sign an honor pledge stating that they did not in any way cheat on the examination, nor did the student share the contents of the examination with anyone for the duration of the examination. A student may not seek assistance from anyone or any source, either in person or from an online source during the examination. Any violation may result in disqualification.**

2. **SCORING: You will receive 6 points for each correct answer, 1 points for each problem left unanswered, and 0 points for each incorrect answer.**

3. **This is a 15-question, short answer exam.**

4. **Some questions call for an exact answer. Utilize sqrt() to express answers involving a square root. Utilize log() to express answers involving a logarithm. Express all answers in lowest terms.**

4. **CALCULATORS AND COMPUTERS ARE NOT PERMITTED. You are permitted to use scratch paper, graph paper, rulers, compasses, protractors, and erasers.**

5. **Figures are not necessarily drawn to scale.**

6. **Enter your answers using the Google form that was provided in your email. The form can also be found on www.maml.org**

7. **Students will submit their answers using an online form. Students and teachers will be notified of the results of the First Level Examination by email when they become available.**

8. **There are no monetary prizes for the First Level Examination.**

9. **Contest materials will be distributed electronically at 3:15pm on November 5, 2020. The contest file will only be sent to those who have signed up to take the exam using the registration form above. Students cannot expect to receive the exam file after 3:15pm on November 5, 2020.**

10. **The window for students to enter answers will be available from 3:30pm until 4:45pm on November 5, 2020. No**

    exceptions will be made.

11. **We are unsure at this time if a Level 2 exam will be offered during the 2020 – 2021 school year.**

12. **If any questions should arise, please contact Michael Curry at currymath@gmail.com**

1. Let r_1 and r_2 be the two values of x which solve the equation below. Compute the smaller value of r_1 and r_2 .

$$x^2 + 7x - 92100 = 0$$

2. The first term of an arithmetic sequence is 13 and the sixth term is 44. Compute the sum of the second, third, fourth, and fifth terms of the sequence.

3. Compute the greatest number of 1-inch by 3-inch by 8-inch books that can be packed in a box measuring 4 inches by 4 inches by 9 inches.

4. Compute the prime p such that $101p + 16$ is a perfect square.

5. Regular hexagon $ABCDEF$ has area 12. Compute the sum of the areas of the 20 distinct triangles whose vertices are three distinct vertices of $ABCDEF$.

6. At time $t = 0$, point A starts at (3,0) and moves around the circle with equation $x^2 + y^2 = 9$, counterclockwise, at a constant rate of 1 revolution every 10 seconds. At the same time, point B starts at (5,0) and moves around the circle with equation $x^2 + y^2 = 25$, counterclockwise, at a constant rate of 1 revolution every 20 seconds. Compute the first positive time t (in seconds) at which A and B are exactly 7 units apart.

7. Given the diagram below, with $A = 60°$, $a^2 - b^2 = 1$, $a^4 - b^4 = 5$. Compute the measure of angle C in degrees.
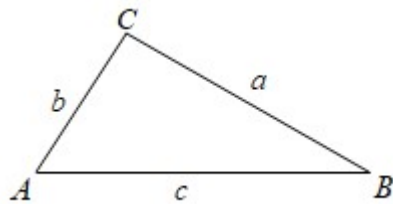
8. Compute the two rightmost digits in the sum: 4! + 8! + 12! + 16! + 20!. Note, $3! = 3 \cdot 2 \cdot 1$.

9. Given $A = (1,4)$, $B = (9,0)$ and P is on the line $y = 2x$ so that $\triangle ABP$ has a right angle at P. Compute the area of $\triangle ABP$.

10. Given a sequence defined as follows: $f(n+1) = 2f(n) + 3$, and $f(5) = 25$, where n is a whole number, compute $f(2)$.

11. **Compute the product of the solutions to the equation below.**

$$x^{\log_4 x^2} = 64x$$

12. **Compute the exact area of a regular octagon circumscribed about a circle of radius one.**

13. **The sum of the first 5n positive integers is 1210 less than the sum of the first 7n positive integers. Compute n.**

14. **The circle $(x-1)^2 + (y-k)^2 = r^2$ where $k > 0$ is tangent to the lines $x = 0$ and $y = 2x$. Compute the sum $k + r$.**

15. **In the binomial expansion of $(x+1)^n$ written in descending powers of x, the ratio of the fifth term coefficient to the sixth term coefficient is 5:6. Compute n.**

END OF CONTEST

Answers:

1. -307
2. 114
3. 5
4. 109
5. 72
6. 20/3
7. 75
8. 44
9. 12
10. 1/2
11. 2
12. $8\sqrt{2}-8$
13. 10
14. $3+\sqrt{5}$
15. 10